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Forum

# Unanswered Questions about Spaced Interleaved Mathematics Practice

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A typical mathematics assignment consists of one or two dozen practice problems relating to the same skill or concept, yet empirical evidence suggests that there is little or no long-term benefit from working more than a few problems of the same kind in immediate succession. Alternatively, randomized experiments in the laboratory and classroom have shown that scores on delayed tests improve markedly when most of the practice problems are arranged so that (a) problems of the same kind are distributed across many assignments spaced weeks apart, and (b) problems of different kinds are interleaved within the same assignment. In this commentary, we describe these math practice strategies and suggest additional lines of research regarding students' and teachers' perceptions of the efficacy and difficulty of these strategies.

*Keywords:* Mathematics, Practice, Spaced, distributed, Inter-leaved

Mathematics students devote much of their effort to practice problems, yet many practice assignments are inefficient or ineffective. For instance, a typical mathematics assignment consists of many problems relating to the same skill or concept, yet evidence suggests that students receive little long-term benefit from working more than several problems of the same kind in immediate succession (e.g., Lyle, Bego, Hopkins, Hieb, & Ralston, 2020). Here we focus on two mathematics learning interventions known as spaced and interleaved practice, and each has proven effective in multiple classroom-based randomized experiments. Spaced practice entails that problems related to the same skill or concept are distributed across multiple assignments, and interleaved practice ensures that problems relating to different skills or concepts are mixed within the same assignment. In this brief essay, we define and illustrate both spaced and interleaved mathematics practice and put forth three avenues of research that might foster their implementation in the classroom. In particular, what are students' and teachers' perceptions about spaced and interleaved practice, and how might these perceptions influence their willingness to use those techniques? Their perceptions matter because the success of an intervention depends partly on whether students and teachers are willing to use it. Too often, the classroom is where promising interventions go to die.

Both spacing and interleaving are instances of a phenomenon known as a *desirable difficulty* (Bjork, 1994)—the focus of this forum. A desirable difficulty is a learning method that, when compared to an alternative, makes practice more difficult while nevertheless improving scores on a subsequent test (e.g., Bjork & Bjork, 2014; Bjork, 2018; Bjork & Bjork, 2019; Bjork & Kroll, 2015; Schmidt & Bjork, 1992). For example, students who wish to learn Spanish vocabulary might find that repeatedly reading through a list of items (CAT–GATO) is easier than using flashcards to test themselves (CAT–?) before looking at the correct response (GATO), but self-testing produces superior test scores (e.g., Butler & Roediger, 2007; Carrier & Pashler, 1992; Roediger & Karpicke, 2006). That difficulties can be desirable is not intuitive. In fact, many people mistakenly assume that the degree of fluency achieved during practice is a good marker of a strategy's long-term efficacy (Bjork, Dunlosky, & Kornell, 2013). Indeed, many difficulties are undesirable in that they impede not only practice performance but also test scores, as might be true for students who do homework while watching television.

## Spaced and Interleaved Mathematics Practice

In nearly every mathematics textbook, the material is divided into many short lessons, and each lesson is followed by a set of practice problems devoted to that lesson. For instance, a lesson on circumference is typically followed by a set of circumference problems. Although this kind of assignment often includes problems that require slightly different strategies, such as using

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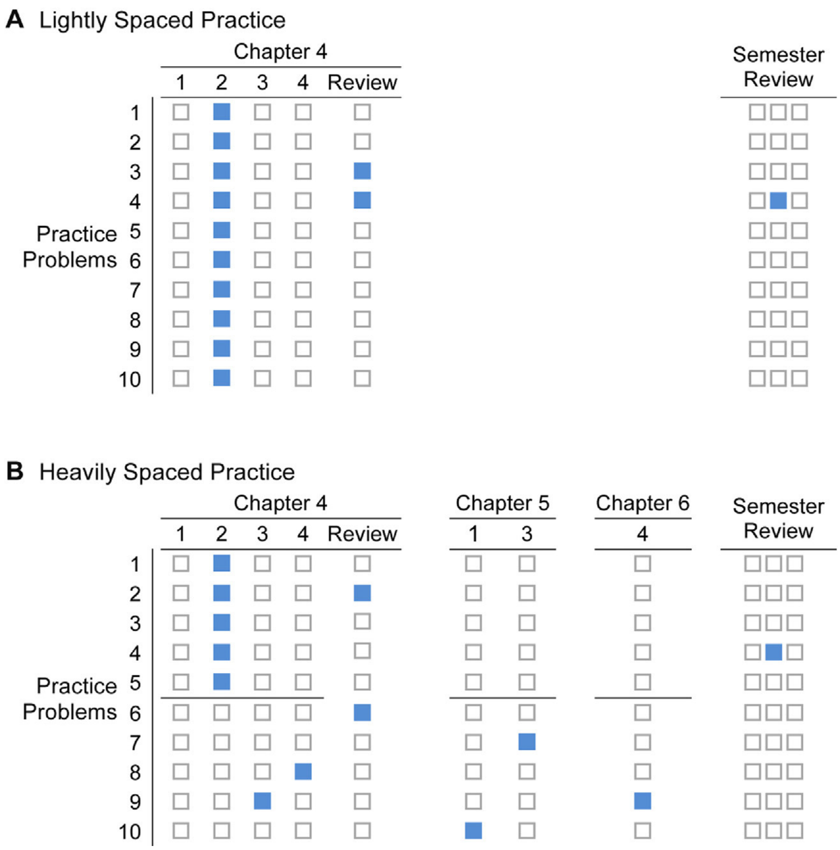


**Figure 1.** Spaced mathematics practice. In this hypothetical illustration, each blue square represents a unique circumference problem, and unfilled gray squares represent problems unrelated to circumference. Each chapter is divided into lessons, and Lesson 2 of Chapter 4 is about circumference. The diagram shows only those assignments that include a circumference problem (i.e., blue square). In most mathematics texts, problems of a particular kind are heavily concentrated in a single assignment (Panel A). In this example, 10 of the 13 circumference problems appear in the assignment following the lesson on circumference. Alternatively, a greater degree of spaced practice is achieved by rearranging problems so that problems of the same kind are distributed more thinly across more assignments (Panel B).

the radius to find the circumference, or using the circumference to find the radius, every problem is nonetheless a problem about circumference. To be sure, most mathematics textbooks also provide periodic review assignments that span multiple topics, but even these assignments are usually divided into small blocks of related problems. For instance, a chapter review assignment typically begins with a few problems relating to the first lesson in the chapter, followed by a few problems relating to the second lesson, and so forth. In one recent review of popular U.S. mathematics textbooks, about 89% of the practice problems were based on the same skill or concept as was the previous problem (Rohrer, Dedrick, & Hartwig, in press).

## The Interventions

In contrast to the usual approach, the practice problems within a course or textbook can be rearranged so that the assignments incorporate two robust learning principles. First, practice problems relating to a specific skill or concept should not be *massed* into a single assignment (or concentrated within only a few assignments) but instead distributed or *spaced* across many assignments that span a long period of time. For instance, a lesson on circumference might be followed by only a handful of circumference problems, with additional circumference problems appearing intermittently throughout the remainder of the text, perhaps with ever decreasing frequency (Fig. 1). Second, practice problems of the same kind should not be *blocked* together but instead arranged so that most are mixed or *interleaved* with different kinds of problems (Fig. 2). Interleaved practice can be provided in various ways. For instance, two consecutive problems might be entirely unrelated, or two consecutive problems might be superficially similar yet fundamentally different, such as a probability problem on sampling with replacement followed by a problem on sampling without replacement.

It might seem that spacing and interleaving are essentially the same strategy, but there is a critical distinction between the two. Spaced practice describes the scheduling of a *single* kind of practice problem, whereas interleaved practice describes the arrangement of *multiple* kinds of practice problems. In fact, it is possible to arrange the practice problems within a textbook or course so that practice is heavily spaced but scarely interleaved – for example, all circumference problems appearing within three blocks spaced one month apart. Still, it is true that a greater degree of interleaved practice guarantees a greater degree of spaced practice.

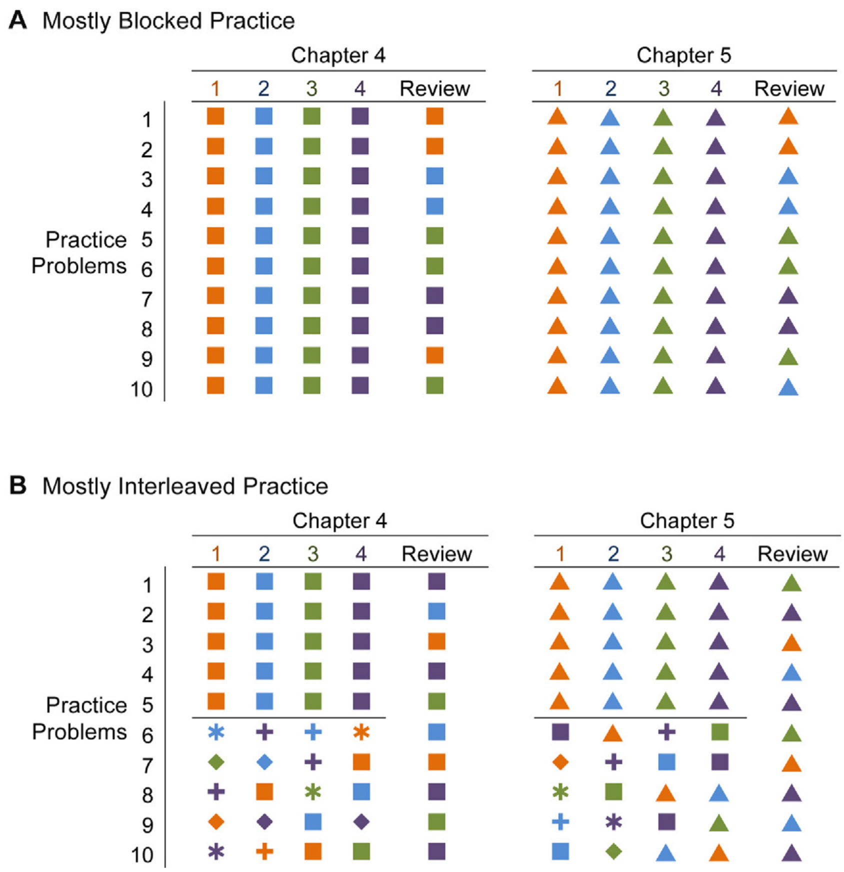


**Figure 2.** Interleaved mathematics practice. In this hypothetical illustration, each symbol represents a unique problem, and each blue square represents a unique circumference problem. Squares represent problems on lessons in Chapter 4, and triangles represent problems on lesson in Chapter 5. All other symbols represent problems from Chapters 1, 2, or 3. Each chapter is divided into lessons, and Lesson 2 of Chapter 4 is about circumference. In most textbooks, problems of the same kind are generally blocked together (Panel A). Alternatively, most problems can be interleaved with different kinds of problems (Panel B). In this example, Lesson 2 of Chapter 4 is followed by an assignment that included five circumference problems (blue squares), one problem from the preceding lesson (orange square), and one problem from each of four lessons from previous chapters.

## Evidence and Rationale

The benefits of spaced and interleaved mathematics practice have been examined in randomized studies in both the laboratory and classroom, and these studies show that spacing and interleaving improve scores on delayed tests. These test benefits of spacing and interleaving—known as the spacing effect and interleaving effect—have been found with a wide variety of materials and procedures, and the test benefits are often long-lasting. For instance, in one recent spacing experiment that was embedded within a college mathematics course, the spacing of practice problems produced higher scores on a test given at the beginning of the following semester, four weeks after the completion of the course (Lyle et al., 2020). In a recent study of interleaving, seventh-grade mathematics students periodically received assignments that provided either mostly blocked or mostly interleaved practice, and the higher dose of interleaving boosted scores on an unannounced test given one month later (Rohrer, Dedrick, & Hartwig, 2020). Other studies of spaced or interleaved mathematics practice are summarized in the aforementioned reports by Lyle et al. and Rohrer et al., respectively.

We should emphasize that evidence also suggests that the interleaving effect is not merely a spacing effect in masquerade, which is to say there is a benefit of interleaving per se. For instance, in one experiment comparing interleaved and blocked practice by fourth grade students, the degree of spacing was equated, and the test scores nevertheless showed a large interleaving effect (Taylor & Rohrer, 2010).

How could interleaving provide benefits above and beyond the benefits of spacing? By one possibility, the interleaving of different kinds of problems prevents students from safely assuming that each problem requires the same strategy as the previous problem, and thus they must instead choose an appropriate strategy on the basis of the problem itself, just as they must do when they encounter a problem while taking an exam or tackling a real-world task (e.g., Kester, Kirschner, & van Merriënboer, 2004). With blocked practice, on the other hand, students often know the appropriate strategy for a problem before they read the problem. In fact, students often know the appropriate strategy for the *first problem* in a block because the block is typically preceded by several worked examples and perhaps a heading such as "Circumference." By contrast, interleaved practice provides students with an opportunity to identify the features of a problem that indicate which strategy is appropriate, which is to say that interleaving teaches students to make the kinds of discriminations that are ubiquitous in nearly every mathematics course. For example, a story problem relating to an incline might lead a middle school student to consider a number of seemingly appropriate strategies (e.g., slope formula, similar triangles, and Pythagorean theorem), but most of the strategies might lead to a dead end. Similarly, a psychology statistics student is unlikely to learn which kind of statistical procedure (e.g., repeated-measures *t*-test) is appropriate for a given scenario if all of the practice problems relating to the procedure are blocked into a single assignment that immediately follows the lesson on that procedure. In simplest terms, interleaved practice provides students with an opportunity to both choose and use a strategy, which is exactly what students are expected to know. By this account, spacing improves long-term retention, and interleaving improves the ability to pair each kind of problem with an appropriate strategy.

Regardless of the underlying mechanism, the larger point for practical purposes is that spaced and interleaved mathematics practice boost scores on delayed tests. These benefits have been observed under a wide variety of ecologically valid conditions using educationally meaningful test delays, and the effect sizes are often large–especially for interleaving. For these reasons, spaced and interleaved mathematics practice has been promoted in outlets intended for learning researchers, practitioners, and laypeople (e.g., Carpenter, 2014; Dunlosky, 2013; Kang, 2016; Pan, 2015; Roediger & Pyc, 2012; Willingham, 2014).

## Potential Barriers to Implementation

If spaced and interleaved mathematics practice are effective, why are both strategies used infrequently in the classroom? One possibility is that spaced interleaved practice is not readily available because massed blocked practice predominates most mathematics textbooks, as detailed above. It is not clear whether this arrangement is by design or by default, but the scarcity of spaced interleaved practice can be remedied. For instance, the creators of mathematics textbooks and other learning materials can provide a greater degree of spaced and interleaved practice by rearranging a portion of the practice problems as part of the revisions in the next edition. Short of that, teachers can provide spacing and interleaving by creating assignments that include, say, one practice problem from each of a dozen earlier assignments in the textbook. (One caveat, though, is that students cannot space their practice of concepts introduced late in the course unless this end-of-course material appears in a subsequent math course.)

Apart from its availability, the implementation of spaced interleaved mathematics practice might be hindered by the beliefs and perceptions of students and teachers. They are the ultimate arbiters of an intervention's utility because they decide whether to use an intervention, and this decision is based partly on their perceptions, accurate or otherwise. In light of these potential obstacles, we suggest three avenues of research.

### Do People Believe that Spaced and Interleaved Mathematics Practice are Effective?

Students and teachers are more likely to use a learning method if they believe it is useful, and thus their beliefs about efficacy need to be understood. Indeed, many studies of *non-mathematics* learning have shown that people often fail to appreciate the benefits of spacing and interleaving. For example, in one laboratory study of category learning, most of the subjects indicated that blocked practice was more effective than interleaved practice even though they had just completed an experiment in which nearly all of them benefitted from interleaving (Kornell & Bjork, 2008). Similarly, when this same category learning task was described in surveys given to students and teachers, most mistakenly predicted that blocking would be the superior strategy (McCabe, 2011; Morehead, Rhodes, & DeLozier, 2016). With regard to spacing, however, beliefs about efficacy may depend on the scenario because the data suggest that students and teachers recognize the benefits of spacing in some contexts (Morehead et al., 2016; Susser & McCabe, 2013) but not in others (e.g., Kornell, 2009; Wissman, Rawson, & Pyc, 2012). As previously alluded to, the failure to appreciate the test benefit of spacing and interleaving might arise because students and teachers falsely believe that a learning strategy that impedes practice must be an inefficient or ineffective learning strategy (Bjork et al., 2013).

The failure to appreciate the benefits of spacing and interleaving might hold for mathematics learning as well, though this possibility has not been fully explored. In one recent study, though, students were asked to create hypothetical math schedules with the aim of maximizing exam scores, and most subjects created schedules that provided only a small degree of spacing and interleaving (Hartwig, Rohrer, & Dedrick, 2020). Yet it is unknown whether math teachers would show a similar neglect for spaced and interleaved practice. Furthermore, beliefs about best practice might vary by circumstance—such as the amount of time until the test or the number of concepts to be tested. More research is needed to understand students' and teachers' beliefs about spaced and interleaved mathematics practice, as well as their rationale for those beliefs, if we are to debunk any misconceptions they might have regarding effective learning practices.

### Do People Believe that Spaced and Interleaved Mathematics Practice is Difficult?

Even if students and teachers believe that an intervention is effective, they might not use it if they find it difficult or otherwise unacceptable. This concern is especially relevant to spaced and interleaved practice because both strategies appear to impair practice performance. For instance, studies have found that practice scores are reduced by both spacing (e.g., Lyle et al., 2020) and interleaving (e.g., Taylor & Rohrer, 2010). Anecdotal evidence also suggests that spacing and interleaving slow the completion of practice problems, though it appears that no published studies have tested this possibility. If such slowing did exist, the test benefits of spacing or interleaving would be smaller when measured against the cost of additional time on task.

From a practical viewpoint, though, the difficulty of any learning technique is better judged through the eyes of students and teachers, yet virtually nothing is known about their beliefs about spaced and interleaved mathematics practice. In what appears to be the only relevant study, a small sample of mathematics teachers were asked to compare interleaved and blocked practice on a variety of dimensions, and a majority reported that their students believed that interleaved assignments were "slightly harder" and took "slightly more time" than did blocked assignments (Rohrer, Dedrick, Hartwig et al., 2020). On the other hand, most of these teachers also reported that their students found interleaved practice to be no less likeable than blocked practice, and nearly all indicated that they wished their students' textbook included more interleaved assignments. However, these findings are based on self-report data, and the participating teachers had participated in an experiment comparing interleaved and blocked practice. In brief, it is unclear whether these difficulties affect the willingness of students or teachers to use these strategies.

## Does Spaced and Interleaved Math Practice Affect Student Confidence?

Students' and teachers' willingness to use a learning strategy can also be affected by how well they think they are learning. For instance, if an assignment can be done fluently and quickly, students may feel confident in their learning, and both students and teachers may perceive the employed learning strategy to be effective and worthwhile (Bjork et al., 2013). While such impressions may sometimes be correct, student confidence can be misleading. Indeed, students sometimes feel confident they have satisfactorily learned materials or skills when in fact they have not—a phenomenon known as an illusion of mastery (e.g., Bjork et al., 2013; Schmidt & Bjork, 1992; Son & Simon, 2012).

With mathematics learning, very little is known about students' confidence during or after practice. It is plausible that the spacing and interleaving of mathematics practice would decrease student confidence because the practice is challenging. With spaced practice, problems of the same kind are distributed across assignments spaced apart by weeks or months, and the solution of the problem requires students to retrieve information from long-term memory instead of merely relying on information seen just moments ago. Similarly, when different kinds of problems are interleaved, students must be able to choose an appropriate strategy on the basis of the problem itself. In contrast, when practice problems are massed or blocked, students can often repeat the same strategy many times, without having to recall the procedures from memory or discriminate among strategies. Thus, whereas spaced interleaved practice might reduce confidence, massed blocked practice might increase it, leading students and teachers to prefer the suboptimal strategy of massed blocked practice. Further, massed blocked practice might increase confidence to the point of overconfidence, which itself can be detrimental because it can lead students to quit practicing long before they should. In brief, massed blocked mathematics practice might produce a sense of fluency during practice and confidence in one's learning (perhaps overconfidence), whereas spaced interleaved practice might reduce them. Of course, these speculations are merely conjecture and require further research.

## Final Remarks

If future research does suggest that the perceptions of students and teachers might be barriers to the classroom implementation of spaced and interleaved mathematics practice, the obstacles are not insurmountable. Most importantly, any added difficulty introduced by spacing and interleaving can be attenuated by scaffolding of various kinds. For instance, after seeing a new skill or concept, it might be optimal for students to immediately work several problems in immediate succession, followed by a gradual fade to spaced interleaved practice (as in Fig. 2). Further scaffolding could be provided by presenting a full solution to students immediately after they attempt a problem so that they can correct their solution. In fact, students and teachers should understand that errors made during practice are acceptable and possibly beneficial, as long as students understand their mistakes and can ultimately provide the correct solution. More broadly, both students and teachers should be explicitly taught that difficulties arising during practice do not necessarily mean that practice is suboptimal or inefficient, and this instruction should include tutorials on the implementation and benefits of spaced and interleaved practice. This in turn will require that those who train teachers must themselves understand that spaced and interleaved practice are efficacious, which likely requires continued advocacy by learning scientists.

We fear, however, that continued advocacy might fall on deaf ears. Indeed, spacing remains rare in non-mathematics courses even though learning scientists have been advocating for spacing for at least 40 years (e.g., Bahrick, 1979; Dempster, 1988). Apart from the possible barriers described in this paper, we believe the limited classroom use of evidence-based interventions like spacing is partly because empirical evidence is not highly valued by many of the educators who recommend learning methods and train teachers (e.g., Robinson, Levin, Thomas, Pituch, & Vaughn, 2007; Sylvester Dacy, Nihalani, Cestone, & Robinson, 2011). Against this backdrop, it might be difficult to inspire the kind of support for evidence-based interventions like those that sparked the dramatic improvements in Western medicine over the last century. Doing so, we believe, is the most pressing challenge facing learning scientists.

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## Conflict of Interest

The authors declare that they have no conflict of interest.

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