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Editors' foreword for the special issue honoring William Kaye Estes

In *Some Targets for Mathematical Psychology*, Estes (1975) addressed the progress made by mathematical psychologists. Although, it was meant to be an assessment of the field's progress, Estes, as usual, chose to look to the future rather than to the past. In doing so, he was less sanguine than perhaps most might have expected. His article is actually a riff off the excellent commentary by Krantz, Atkinson, Luce, and Suppes (1974a,b) who wrote "It is easy to point to excellent work in mathematical psychology, past and present. But in retrospect, cumulative progress is less easy to find . . . without apologizing for the past, we do need to ask ourselves whether we can do better in the coming decades. Is it possible that the lack of cumulative progress is partly due to that goal being subordinated to others, such as originality or technical mastery? If so, then that goal needs to be formulated more directly, and seeking it needs to be encouraged" (p. xii–xiv). Estes concurred, "This view is doubtless shared by many investigators in our field, but even if it were not, it would need to be taken seriously since it evidently represents the consensus of four of our most eminent contributors". (p. 264)

Estes' thesis was that the apparent lack of "cumulative progress" noted by Krantz et al. was the result of not having a clearly defined set of goals for mathematical psychology. At the time, computers were being rapidly introduced to the field and some questioned the utility of computer simulations of psychological phenomena. There was some concern that a divide was emerging among those utilizing different tactics to understand psychological phenomena. For Bill, the tactic used to solve a problem or increase understanding was less important than whether the question one asked was theoretically important. Estes advocated a focus on strategy rather than specific method and he provided a set of goals for the field. These goals did not relate to specific research, rather, the goals were broadly stated in a hope that advances toward them would create an identity and relevance for mathematical psychology. The extent to which it can be attributed to Estes' cajoling is unknown, but substantial progress has been made since that 1975 address.

The articles in this special issue in honor of William K. Estes highlight several cogent examples of cumulative progress in the field of Mathematical Psychology. The spectrum of the Estes legacy is so broad in scope that any such anthology must inevitably prove narrow in comparison, but each of the following articles focuses on an important psychological problem, often with connections tracing back to Estes's own research, and follows his precepts concerning scientific strategy.

Estes always encouraged his fellow psychologists to take an ill-defined psychological problem and turn it into a more well

defined mathematical modeling problem. This lesson is central to the goal of evaluating the necessity of quantum principles for understanding psychological phenomena. In *The dynamics of decision making when probabilities are vaguely specified*, Pothos, Shiffrin, and Busemeyer model how a decision to terminate a series of risky decisions is made when it is unknown whether one's outcome has been maximized. By deriving a model from quantum theory and testing it empirically, the clear result is that certain quantum models of decision making can be rejected, and an empirical benchmark with which to compare future models is established.

Marc Howard's contribution, *Mathematical learning theory through time*, traces the evolution of the assumption that internal representations of an item in context fluctuate over time from stimulus sampling theory (Estes, 1950) through the temporal context model. He concludes by reviewing data showing that these ideas are well characterized by neural recordings.

In *Associations and manipulations in the mental lexicon: A model of word-stem completion* Mueller and Thanasuan introduce a model for word-stem completion that is a combination of stimulus sampling theory (Estes, 1950, 1955a,b) along with a strategic generation process.

Worthy and Maddox describe advances to Estes (1950, 1955a,b) learning theory in their contribution, *A comparison model of reinforcement-learning and win-stay-lose-shift decision-making processes: A tribute to W.K. Estes*. They present a model incorporating a reinforcement learning mechanism, wherein the relative value of each option is critical, and a rule based mechanism, wherein the outcome of the previous trial determines performance on the current trial.

Markov models developed by Estes (e.g., 1959) and colleagues, once abandoned in favor of complex process models, are making a comeback perhaps because of the elegance in the simplicity of such models. In *Markovian interpretations of dual retrieval processes*, Gomes, Brainerd, Nakamura, & Reyna present a Markovian interpretation of a class of dual process memory models including mechanisms of direct access, reconstructive recollection, and familiarity. Also in the Markov trend, Chechile and Sloboda delineate the relationship between Markovian models and hazard functions applied to learning and memory in *Reformulating Markovian processes for learning and memory from a hazard function framework*.

Yang, Fific, and Townsend detail the early contributions of Estes to the emerging information processing approach to visual cognition in their overview of the history of mathematical psychology. In their contribution, *Survivor interaction contrast*

wiggle predictions of parallel and serial models for an arbitrary number of processes, they proceed to present an extension of the survivor interaction contrast and demonstrate how this extension can provide a deeper understanding of visual and short term memory search.

Cumulative progress is the hallmark of Roger Ratcliff's contribution to mathematical psychology beginning with his research with Bennett B. Murdock on measuring reaction times (Ratcliff & Murdock, 1976). In *Modeling perceptual discrimination in dynamic noise: Time-changed diffusion and release from inhibition*, Smith, Ratcliff, and Sewell extend the diffusion model to two-choice perceptual discrimination in dynamic noise.

Estes' (1975) stated, "Undoubtedly the elegant formal accomplishments of mathematical psychology are those having to do with models of measurement (see, e.g., Krantz, Luce, Suppes & Tversky, 1971). One motivation for the heavy concentration of attention on measurement has been the intrinsic interest of the mathematical problems and the possibility of finding solutions, always a winning combination." Nonetheless, Estes's own seminal contributions to measurement in psychology typically involved adroit merges of psychometric strategies with original model-based depictions of mental processes. A salient example of this work rests on his introduction of latent constructs to models of learning.

In *Measures of association in contingency space analysis*, Martens, Gertz, Werder, Rymanowski, and Shankar provide novel insight into measuring contingent relationships between behavior and consequences, thereby carrying on a tradition that Estes began back in his stunning papers on stimulus sampling theory (1955a; 1955b).

Estes possessed quite an affinity, as well as talent, for utilizing models as a means of classifying phenomena. Moustafa, Herzallah, and Gluck continue in this train and present *A model of reversal learning and working memory in medicated and unmedicated patients with Parkinson's disease*.

The field of mathematical psychology has become strongly associated with methods for model selection and testing and the application of sound statistical analyses in psychological science. In 1956, Estes highlighted the problems of identifying models based on group averages, and the issue of measuring individual differences remained a focus of his research (e.g., Estes and Maddox, 2005). In *A Bayesian hierarchical mixture approach to individual differences: Case studies in selective attention and representation in category learning*, Bartlema, Lee, Wetzels, and

Vanpaemel present a method for organizing individuals based on their classification behavior.

In his keynote address to the Psychonomic Society, published in 2002, Estes stated "Over more than a half century of experience in research on learning, memory, and decision, I have come to believe that the most substantial and enduring advances have not been in the accumulation of empirical facts or the construction of models, but in the production of fruitful interactions between models and experimental research". The articles in this special issue represent several examples of such advances. Beyond honoring the legacy and contribution of William K. Estes, we anticipate that these articles will assist in providing the foundation upon which to build another 40 years of cumulative progress.

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