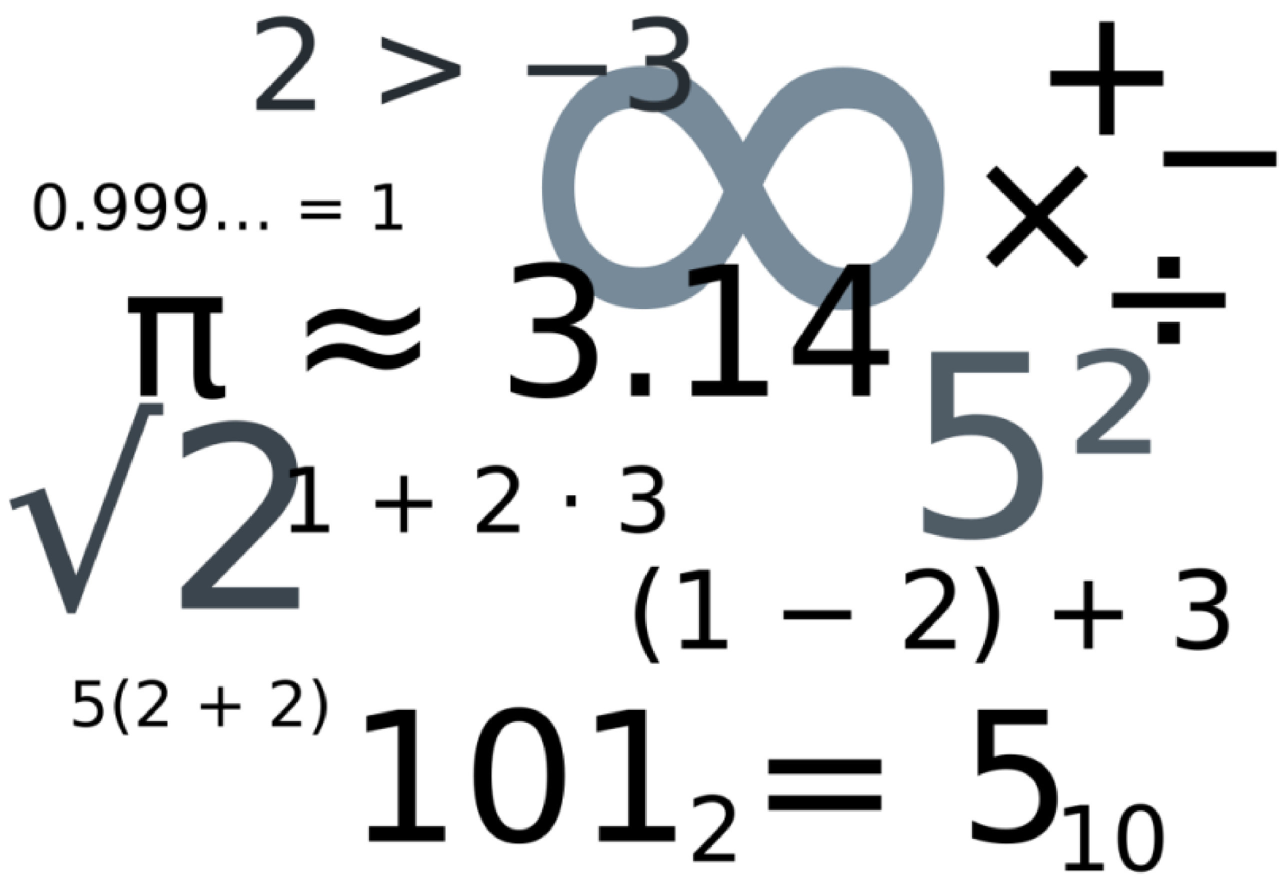
Interleaved Mathematics Practice

Giving Students a Chance to Learn What They Need to Know



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Overview

This guide describes a learning method that works in any math course.

The method is called interleaved practice, and it's easy to use. It doesn't require teachers to change how they teach, and students can use it in class or at home, with or without computers. And it's free.

Interleaved practice is also backed by science. It has proven superior to the business-as-usual approach in classroom-based randomized control trials—the gold standard of evidence.

Interleaved practice is simply a set of problems mixed in a certain way.

Practice problems are interleaved if the problems are arranged so that consecutive problems cannot be solved by the same strategy. For example, if one problem is solved by finding the area of a circle, the next problem requires a different strategy, such as solving an inequality.

Whereas a typical assignment consists of problems that can be solved by the same strategy, which usually lets students know the strategy for each problem before they even read it, interleaved practice forces students to choose a strategy on the basis of the problem itself, just as they must do when they work problems on cumulative exams and other high-stakes tests. Put another way, interleaved practice gives students a chance to learn what they need to know.

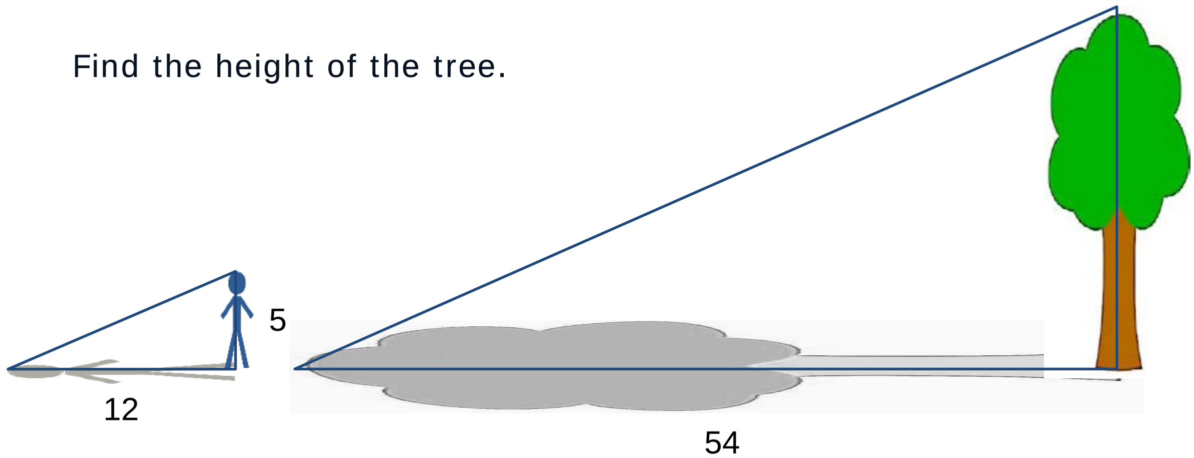
This guide includes everything students and teachers need to know about interleaved practice. We'll start by explaining why it's needed.

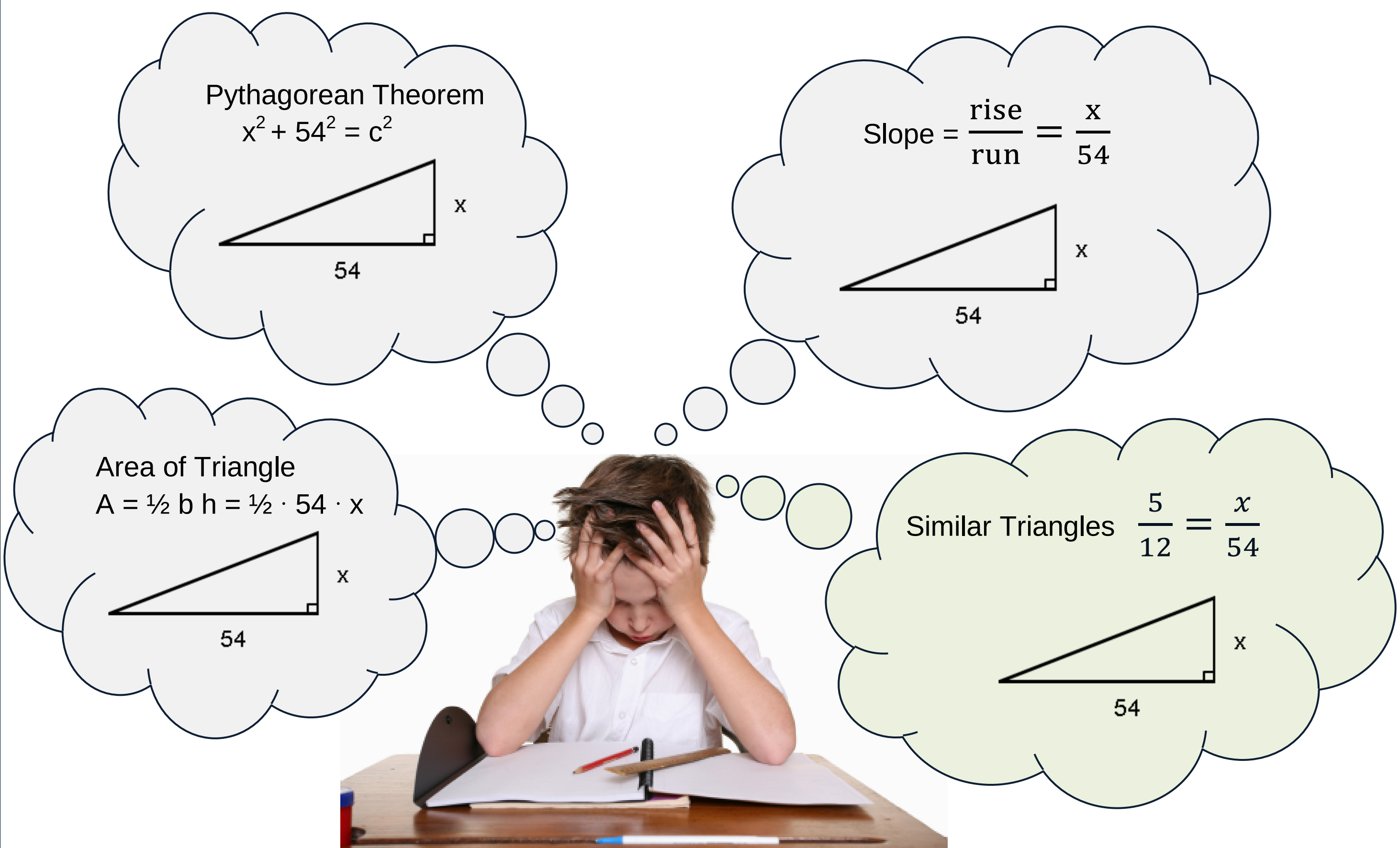
Choosing a Strategy

The solution of a math problem begins with the choice of a strategy.

To solve a problem, students must first choose an appropriate strategy—often the hardest step. Students see many strategies, and a strategy that seems useful might turn out to be useless. In the problem below, for instance, several strategies seem relevant, but only one is helpful.

Find the height of the tree.





Another Strategy Choice Dilemma

Problems that look alike sometimes require different strategies.

Another reason why it's often hard to choose a strategy is that problems that look alike cannot always be solved by the same strategy. Students face this challenge in every math course from Arithmetic to Calculus.

Problem	Strategy
Arithmetic	
Ed ate 5 cookies, and he now has 9. How many did he start with?	Addition
Ed ate 5 cookies, and he began with 9. How many does he have?	Subtraction
Algebra	
Solve. $x - 3x + 2 = 0$	Group x terms on one side
Solve. $x^2 - 3x + 2 = 0$	Factor
Geometry	
Find the length of the line segment with endpoints (1,1) and (5, 4)	Pythagorean Theorem
Find the slope of the line segment with endpoints (1,1) and (5, 4)	Rise / Run
Trigonometry	
For $\triangle XYZ$, find *x* if $\angle X = 120°$, y = 3, and z = 5.	Law of Cosines
For $\triangle XYZ$, find *x* if $\angle X = 120°$, y = 3, and $\angle Y = 50°$.	Law of Sines
Calculus	
$\int x(e + 1)^x \, dx$	Integration by Parts
$\int e(x + 1)^e \, dx$	U-Substitution

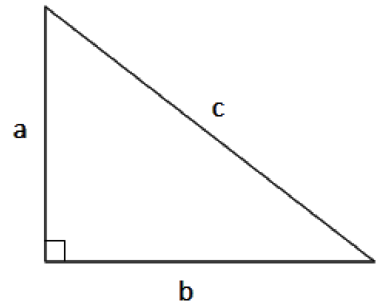
Blocked Practice

Most assignments don't require students to choose a strategy.

A typical assignment includes a block of problems that are related to the same concept, so students usually know an appropriate strategy for each problem before they read the problem.

For example, the assignment below is entitled *The Pythagorean Theorem*, which means that students know the strategy for each problem before they begin the first problem.

However, if one of these problems appeared on a cumulative exam, students would need to infer from the problem itself that they should use the Pythagorean Theorem. This choice of strategy is harder than it might seem at first because none of the problems include helpful terms such as *triangle*, *hypotenuse,* or *Pythagorean Theorem*.



Blocked Practice

NAME____ DATE __________ PERIOD ___

3-5 The Pythagorean Theorem

Glencoe McGraw-Hill
Math Connects Course 3 ©2017

1. What is the length of a diagonal of a rectangular picture whose sides are 12 inches by 17 inches?

$$12^2 + 17^2 = x^2$$

2. Ross has a rectangular garden in his backyard. He measures one side of the garden as 22 feet and the diagonal as 33 feet. What is the length of the other side of his garden?

$$22^2 + x^2 = 33^2$$

3. Troy drove 8 miles due east and then 5 miles due north. How far is Troy from his starting point?

$$8^2 + 5^2 = x^2$$

An Illusion of Mastery

Blocked practice deceives students and teachers.

Many students zip through a blocked assignment and then mistakenly conclude that they can solve the problems when, in fact, they haven't learned to solve the problems without knowing the strategy in advance. This false belief is called an *illusion of mastery*.

The illusion is shattered when these students see the same kind of problem on a cumulative test but cannot choose an appropriate strategy. Although these students often attribute their failure to test anxiety, a simpler explanation is that too many of their practice problems were blocked, and they therefore never learned to solve problems without knowing the strategy in advance.

In short, blocked practice provides students with a crutch. If students don't learn to solve problems without it, they will struggle during a test when their crutch is snatched away.

The Prevalence of Blocked Practice

A majority of textbooks and workbooks provide mostly blocked practice.

One recent analysis of six popular middle school math textbooks found that, on average, more than 80% of the practice problems were blocked. The percentage of blocked practice is even greater in most consumable workbooks and downloadable worksheets.

Even most review assignments include blocked practice.

Although most math textbooks include chapter review problems, these assignments usually consist of small blocks of problems—for example, a few problems on Lesson 5-1, followed by a few problems on Lesson 5-2, and so forth. We call these mini-blocks.

Mini-blocks also comprise many of the assignments described as *spiral review* or *mixed review*. For example, the Mixed Review assignment below includes three mini-blocks.

Blocked Practice

Pearson Prentice Hall
Algebra 1 ©2011

Mixed Review

Simplify each expression.

81. $bc^{-6}b^{-3}$
82. $(a^2b^3)(a^6)$
83. $9m^3(6m^2n^4)$
84. $2t(-2t^4)$

Find the slope of the line that passes through each pair of points.

85. (0, 3), (4, 0)
86. (2, –5), (3, 1)
87. (–3, 6), (1, 0)
88. (0, 0), (1, –9)

Write each fraction in simplest form.

89. $\frac{5}{20}$
90. $\frac{124}{4}$
91. $\frac{6}{15}$
92. $\frac{5xy}{15x}$
93. $\frac{3ac}{12a}$

The Remedy

Interleaved practice gives students a chance to choose a strategy.

When practice problems are arranged so that consecutive problems cannot be solved by the same strategy, students are forced to choose a strategy on the basis of the problem itself. This gives students a chance to both *choose* and *use* a strategy.

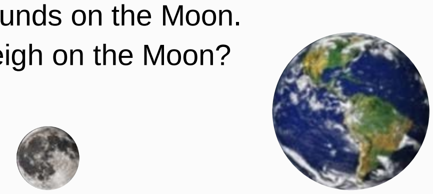
Interleaved practice doesn't require students to solve *extra* problems.

Interleaved practice can be added to a textbook or course without adding new problems. For instance, a few problems can be removed from each blocked assignment, and these problems can be combined to create interleaved assignments like this one.

Interleaved Practice

1. Rhea weighs 72 pounds on Earth, but she would weigh only 12 pounds on the Moon. If her brother weighs 126 pounds on Earth, how much would he weigh on the Moon?

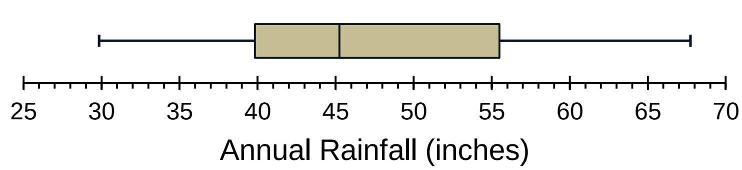
$$\frac{72}{12} = \frac{126}{x} \qquad x = 21$$



2. Beth wakes before dawn and randomly grabs two socks from a drawer without turning on a light. The drawer has 3 red socks and 2 blue socks. Find the chance that she grabs two red socks.

$$\frac{3}{5} \times \frac{2}{4} = \frac{6}{20} = \frac{3}{10}$$

The box plot shows Tampa's annual rainfall for the last 40 years.



3. Find the median annual rainfall to the nearest inch.

45

4. In approximately how many of the last 40 years did Tampa receive at least 55 inches of rain?

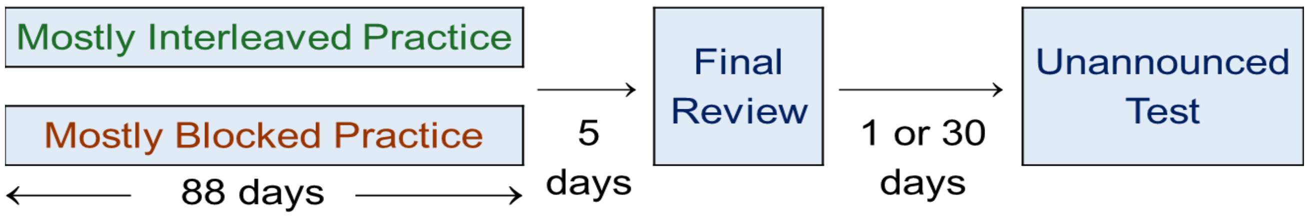
25% of 40 = 10

Evidence for Interleaved Practice

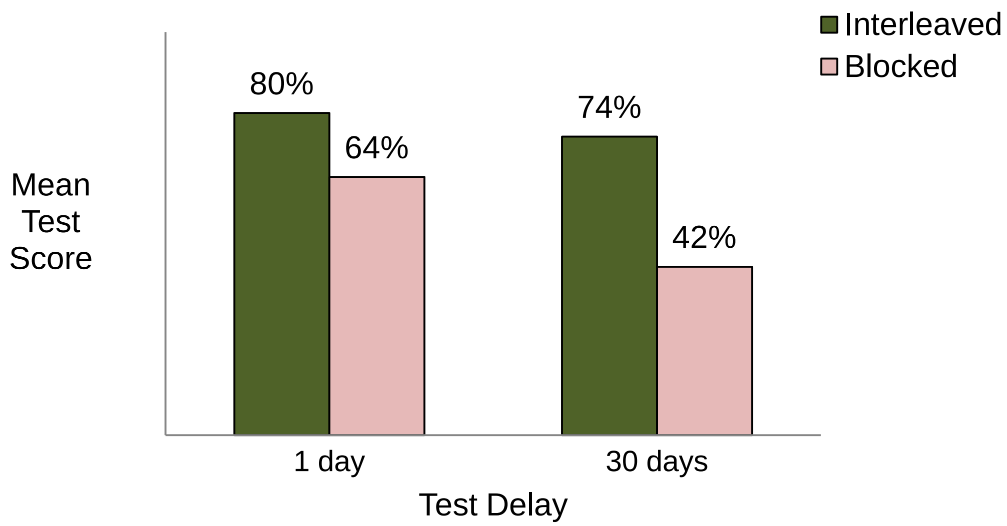
Interleaved practice works.

In several randomized control studies, students who received mostly interleaved practice scored higher on a final test than did students who received mostly blocked practice.

In one of these studies, for instance, seventh-grade students received worksheets for 3 months, and the worksheets were organized so that the problems relating to a particular concept were either blocked together or interleaved with other problems. Yet every student saw the *same* problems—only the schedule varied. After the practice phase, every student saw a final review with one problem of each kind. This was followed 1 or 30 days later by an unannounced test.



More interleaved practice led to higher test scores, especially after a 30-day delay.



Rohrer, Dedrick, and Stershic (2015)

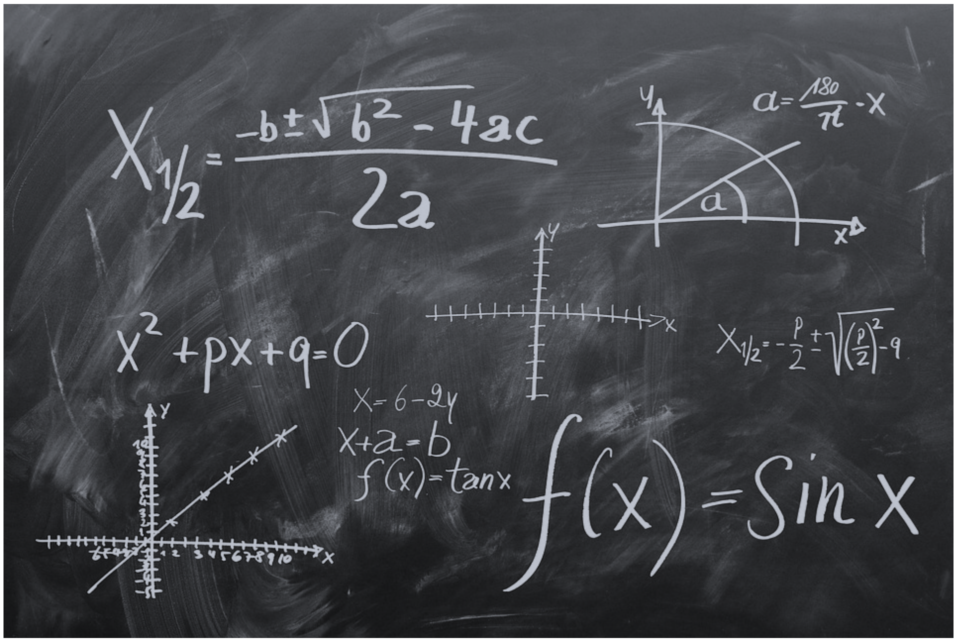
Where does one find interleaved practice assignments?

Some textbooks and workbooks provide lots of interleaved practice. If these resources aren't available, students and teachers can do the following:

1. Create an interleaved assignment by choosing one problem from each of several blocked assignments in a typical textbook or workbook. For instance, the assignment might include p. 37 #19, p. 117 #21, p. 156 #3, and so forth.

2. Use the review assignments in the students' textbooks and workbooks. Although review assignments usually include mini-blocks (see page 7), these assignments provide at least some interleaved practice.

3. Find interleaved worksheets and practice tests on the Internet. For instance, a web search for "mixed review mathematics worksheets" returns a list of websites with freely downloadable interleaved assignments. And interleaved practice tests can be found on the websites of companies and organizations that create mathematics assessments, including *PARCC* and *Smarter Balanced*.



Caveats

1. Students need at least some blocked practice.

Some blocked practice is useful, especially when students encounter a new concept or skill. How much interleaved practice is enough? The ideal amount depends on the student and the material, but studies suggest that at least a third of the practice problems should be interleaved.

2. Students must see the solutions and correct their errors.

Interleaved practice must be followed by informative feedback. While students can whiz through a blocked assignment by repeating the same procedure, an interleaved assignment includes a variety of problems, including ones students haven't seen recently. So students should see the solutions, correct their errors, and have a chance to ask questions.

3. Interleaving improves test scores only when tests are cumulative.

Interleaved practice probably won't boost scores on tests covering only the most recent material. For example, if a unit on proportions is followed by a test consisting only of proportion word problems, students will likely know the relevant strategies before they begin the test. For the same reason, though, non-cumulative tests are not a good indicator of student proficiency.

The Takeaway

Interleaved practice isn't flashy. It doesn't offer the bells and whistles of computer animation, and it doesn’t promise a quick fix.

Yet interleaved practice is inspired by well-established scientific principles, and it has produced large benefits in classroom-based randomized control trials.

Interleaved practice also makes sense. Students must learn how to both *choose* and *use* a strategy because that is what they must do on cumulative exams and other high-stakes tests. Simply put, interleaved practice gives students a chance to learn what they need to know.

References

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Notes

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